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Response to Kowalczyński on Tachyons

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I answer and briefly comment upon a paper on tachyons by J. K. Kowalczyński. Suitable answers are already contained in the recent literature about "extended relativity" (ER), apparently unknown to that author. My answer is threefold. (1) About causality: No paradoxes can be sensibly discussed without studying in detail the tachyon-exchange dynamics; but, once one knows tachyon mechanics, the solution of the paradox is straightforward. As an example, I exploit and solve the Tolman-Regge paradox. (2) About superluminal "frames" and "transformations": I agree that (as I have noted elsewhere) in four dimensions such language is unfortunate; it was borrowed from two dimensions, where it is completely justified. Formulations in terms of a new language can be found in my recent papers on ER. (3) The statement that the pseudo-Euclidean space-time is a particular Riemannian manifold is wrong. It is *pseudo*-Riemannian, or Lorentzian. When dealing with tachyons the difference between pseudo-Riemannian and Riemannian is essential.

1. PRELIMINARY COMMENT

Since Kowalczyński (1984) criticizes, among the others, some papers by me or my co-workers, a brief answer is necessary. I shall comment on three points: (1) Causality for tachyons (answering his Section 2); (2) superluminal "frames" and "transformations" (answering his Section 4); and (3) the claim that the pseudo-Euclidean space-time is a particular case of Riemannian space (answering his Appendix B).

Before going on, however, I first observe that Kowalczyński, when criticizing "extended relativity" (ER)—which is of course a *developing* theory—refers mainly to a paper written more than 10 years ago (Recami and Mignani, 1974). Further, Kowalczyński quotes some papers (e.g., Basano, 1977, 1980), but not the rebuttals that immediately followed them

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(e.g., Recami and Pavšič, 1978; Maccarrone and Recami, 1980b). The interested reader is referred to the more recent literature on the subject: for instance, Caldirola and Recami (1980), Maccarrone and Recami (1980a, 1982, 1984), Barut *et al.* (1982), Maccarrone *et al.* (1983), and Smrz (1984). Such papers already answer Kowalczyński's objections.

2. ANSWER ABOUT CAUSALITY

2.1. Introduction

Since I have already discussed the issue of causality not only in Caldirola and Recami (1980), but also, more thoroughly, in a recent review article (C. F. Recami 1984a, b, c), I recall only the following. It is impossible to discuss properly the paradoxes connected with the exchange of tachyons by ordinary bodies without having previously investigated (a) the problem of tachyon localization—in fact, tachyons are not localizable in ordinary space (Barut *et al.* 1982; Recami and Maccarrone 1983); (b) the details of tachyon mechanics, e.g., for the case of two-body interactions via tachyon exchange [as explicitly done already in Maccarrone and Recami (1980a)]. Both points were missed by that author.

Having exploited tachyon dynamics [see Maccarrone and Recami (1980a), hereafter called MR], it becomes simple to solve the ordinary causal paradoxes, at variance with Kowalczyński (1984, p. 30). For brevity, I confine myself here to analysing the Tolman paradox. I use of course the "third postulate" of special relativity, i.e., the Stückelberg-Feynman-Sudarshan "switching principle" (Recami, 1978, 1979; Caldirola and Recami, 1980; Recami and Rodrigues, 1982; Pavšič and Recami 1982; Schwartz, 1982). A complete discussion of the causality issue will be found in the forthcoming review paper.

2.2. Solution of the Tolman–Regge Paradox

The oldest paradox is the "antitelephone" one, originally proposed by Tolman (1917; see also Bohm, 1965) and since reproposed by many authors.

Let us refer to its most recent formulation (Regge, 1981), and spend some care in solving it, since it was misunderstood by Kowalczyński and since it is the kernel of many other paradoxes.

2.2.1. The Paradox

In Figure 1 the axes t and t' are the world lines of two devices A and B, respectively, able to exchange tachyons and moving with constant relative speed u ($u^2 < 1$). According to the terms of the paradox (Figure 1a), A sends tachyon 1 to B [in other words, tachyon 1 is supposed to move forward

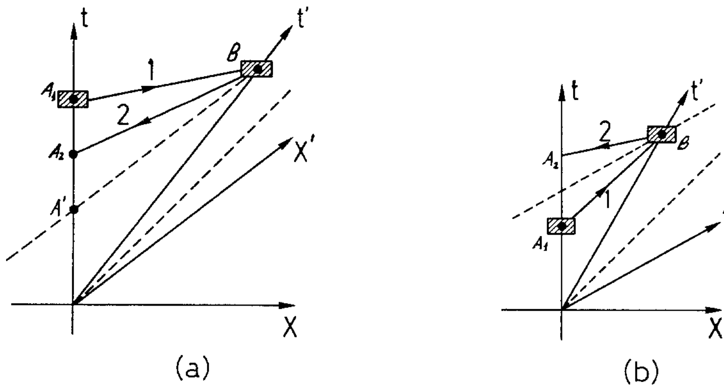

Fig. 1. (a) The Tolman–Regge paradox. For a clear solution, see the text. (b) The actually process, according to the requirements of tachyon mechanics (see test), when the second body B is supposed to receive tachyon 1 and soon after emit tachyon 2 toward A.

in time with respect to (w.r.t.) A]. The apparatus B is constructed so as to send back a tachyon 2 to A as soon as it receives a tachyon 1 from A. If B has to *emit* (in its rest frame) tachyon 2, then 2 must move forward in time w.r.t. B, that is, its world line BA_2 must have a slope smaller than the x'-axis slope BA' (where $BA' \| x'$); this means that A_2 must stay above A'. If the speed of tachyon 2 is such that A_2 falls between A' and A_1, it seems that 2 reaches A (event A_2) *before* the emission of 1 (event A_1). This appears to realize an antitelephone.

2.2.2. The Solution

First of all, since tachyon 2 moves backward in time w.r.t. A, the event A_2 will appear to A as the emission of an antitachyon $\bar{2}$. The observer "t" will see its own apparatus A (able to exchange tachyons) emit successively toward B the antitachyon $\bar{2}$ and the tachyon 1.

At this point, some supporters of the paradox [overlooking tachyon kinematics, as well as equations (11) in MR] would say that the description forwarded by observer "t" can be orthodox, but then the device B is no longer working according to the premises, because B is no longer emitting a tachyon 2 on receipt of tachyon 1. Such a statement would be wrong, however, since the fact that "t" sees an "intrinsic emission" at A_2 does not mean that t' will see an "intrinsic absorption" at B. On the contrary, we are just in the case of Section 5 in MR: intrinsic emission by A, at A_2, with $\mathbf{u} \cdot \mathbf{V}_{\bar{2}} > c^2$, where $\mathbf{u}$ and $\mathbf{V}_{\bar{2}}$ are the velocities of B and $\bar{2}$ w.r.t. A, respectively; so that *both* A *and* B suffer an intrinsic *emission* (of tachyon 2 or of antitachyon $\bar{2}$) in their own rest frames (cf. also Maccarrone and Recami, (1980b)).

But the terms of the paradox cheat us even more, and *ab initio.* In fact, Figure 1a makes it clear that, if $\mathbf{u} \cdot \mathbf{V}_{\bar{2}} > c^2$, then for tachyon 1 *a fortiori* $\mathbf{u} \cdot \mathbf{V}_1 > c^2$, where $\mathbf{u}$ and $\mathbf{V}_1$ are the velocities of B and 1 w.r.t. A. Due to Section 5 of MR, therefore, observer "t'" will see B intrinsically *emit* also tachyon 1 (or, rather, antitachyon $\bar{1}$). In conclusion, the proposed chain of events does not include any tachyon absorption by B.

For body B to *absorb* tachyon 1 (in its own rest frame), the world line of 1 ought to have a slope larger than the x'-axis slope (see Figure 1b). Moreover, for body B to *emit* ("intrinsically") tachyon 2, the slope of 2 should be smaller than the x'-axis slope. In other words, when the body B, programmed to emit 2 as soon as it receives 1, actually does so, the event A_2 regularly happens *after* A_1 (cf. Figure 1b).

2.2.3. The Moral

The moral of the story is twofold: (1) one should never *mix together* the descriptions of one phenomenon yielded by different observers, otherwise, even in ordinary physics, one would immediately meet contradictions: in Figure 1a, e.g., the direction of motion of 1 is assigned by A and the direction of motion of 2 is assigned by B; this is illegal; (2) when proposing a problem about tachyons, one must comply (Caldirola and Recami, 1980) with the rules of tachyon kinematics (Maccarrone and Recami, 1980a), just as when formulating an ordinary problem one must comply with the laws of ordinary physics (otherwise the problem in itself is wrong).

Most of the paradoxes proposed in the literature suffer the above shortcomings.

Notice that, in the case of Figure 1a, neither A nor B regards event A_1 as the cause of event A_2 (or vice versa). In the case of Figure 1b, on the contrary, both A and B consider event A_1 to be the cause of event A_2: but in this case A_1 does chronologically *precede* A_2 for both observers, in agreement with the relativistic covariance of the law of retarded causality (Recami, 1978, 1979; Recami and Rodrigues, 1982).

Kowalczyński (1984, p. 31) maintains that the "emission of a negative quantity" is distinguishable from the "absorption of a positive quantity": on the contrary, $(-) \cdot (-) = (+) \cdot (+)$, also according to the Stückelberg-Feynman-Sudarshan "switching procedure" itself (Caldirola and Recami, 1980).

3. SUPERLUMINAL "FRAMES" AND "TRANSFORMATIONS"

In a "model theory" of ER in two dimension (Recami and Mignani, 1974), superluminal (as well as subluminal) frames exist; and superluminal (as well as subluminal) Lorentz transformations exist. When we tried to generalize ER to more dimensions, we kept the same terminology, to remind us constantly of *the aim* of looking for suitable conditions under which superluminal transformations actually exist. I agree, however, that *in four dimensions* such a language is unfortunate. In fact, it was shown long ago (Gorini, 1971, and references therein) that *no* superluminal Lorentz transformations exist in four dimensions that are real, linear, and meet all the other ordinary requirements, as pointed out also by Kowalczyński [cf. Kowalczyński, 1984, Section 4, e.g. the beginning of p. 48 and p. 52).

The whole main project of ER has been to look for the "minimal departures" from ordinary special relativity [temporarily passing, for instance, to an auxiliary six-dimensional Minkowskian space-time $M(3,3)$] which allow an ER for tachyonic frames to be constructed in analogy to the one so nicely built up in $M(1,1)$. Recall that introducing imaginary quantities is equivalent to increasing the number of dimensions (Maccarrone and Recami, 1982a,b).

I shall go even further: Since a real, linear transformation SLT: $dx_\mu \to dx'_\mu$ for tangent vectors, changing $ds^2 \to -ds^2$, does not exist in four dimensions (see, however, Section 4 below), superluminal real transformations $\mathcal{T}: x_\mu \to x'_\mu$ mapping points of M_4 into points of M_4 *cannot* exist in $M_4 \equiv M(1,3)$ as well. Otherwise, from the existence of a point-to-point superluminal transformation $\mathcal{T}: x_\mu \to x'_\mu$, the existence of a real, linear transformation $SLT: dx_\mu \to dx'_\mu$ for the tangent vectors, carrying ds^2 into $ds'^2 = -ds^2$, would follow too (Recami, 1984a,b,c; Smrz, 1984; Rindler, 1966), contradicting Gorini's no-go theorem. In fact, when trying to go back from the auxiliary $M(3,3)$ to four dimensions, we interpreted the so-called "superluminal transformations" as mapping, for example, points into cones, and not points into points (Barut *et al.*, 1982; Maccarrone and Recami 1982a,b, 1984). Therefore, such "transformations" in four dimensions are expected to be *mappings* carrying manifolds of M_4 into manifolds of (the same) M_4.

A discussion of those superluminal mappings can be found in Maccarrone and Recami (1982b), where, answering Kowalczyński's Appendix A in advance, the problem of the real slices of $M_6 \equiv M(3,3)$ is also faced, (without forgetting the language problems posed by keeping the expression "superluminal transformations" even in four dimensions).

4. REBUTTAL OF APPENDIX B

Kowalczyński in his Appendix B regards pseudo-Euclidean space-time M_4 as a particular case of Riemannian space. Actually Minkowski space-time, like the space-times of general relativity, is known to be Lorentzian, i.e., pseudo-Riemannian. As a consequence, the theorems of Riemannian geometry apply to general relativity, or to M_4, only under precise, restrictive, particular conditions (cf. Møller 1962, Sachs and Wu, 1980).

Other points of that Appendix B are already discussed at the beginning of Section 5 in Maccarrone *et al.* (1982) and at the beginning of Section 5 and in the Appendix in Maccarrone and Recami (1982a,b, 1984). For instance, let us recall that in the complex case (e.g., when using the "Euclidean" metric) one has to define $ds^2 \equiv \langle dx|\overline{dx}\rangle$.

When dealing with tachyons it becomes *essential* to distinguish the pseudo-Riemannian from the Riemannian geometry. In the former (in contrast with the latter) antiorthogonal transformations *a priori* exist that carry $ds^2 \to -ds^2$. For instance, in the simple, two-dimensional case, an antiorthogonal transformation is given by the following (e.g., Recami, 1984a,b,c, and references therein):

$$dt' = \pm\frac{dt - U\,dx}{(U^2-1)^{1/2}} \equiv \mp\frac{dx - u\,dt}{(1-u^2)^{1/2}}$$

$$dx' = \pm\frac{dx - U\,dt}{(U^2-1)^{1/2}} \equiv \mp\frac{dt - u\,dx}{(1-u^2)^{1/2}} \qquad (u \equiv 1/U)$$

with $u^2 < 1$ and $U^2 \equiv 1/u^2 > 1$.

My remarks on Appendix B also apply to footnote 19 at p. 41 in Kowalczyńsky (1984).

Let me conclude by correcting the conclusions of that author as follows: Real, linear SLTs meeting all the ordinary requirements of special relativity are known not to exist in four dimensions; the problem, of 15 years standing, is then to find a physically meaningful, "minimal" enlargement of special relativity that allows us to build up an ER in mor dimensions similar to the one already developed in two dimensions. Our hope in so doing is to reproduce the quantum behavior at a geometrical, classical level (Recami, 1979, 1984a,b,c).

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NOTE ADDED IN PROOF

The most recent (and relevant) information may be found in Recami, E. (1987) *Foundations of Physics*, **17**, 239 and, particularly, in Recami, E. (1987) *Rivista del Nuovo Cimento*, **9**, 1.

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